Introduction to Sustainable Urban Development Planning

LECTURE 11

MULTICRITERIA DECISION ANALYSIS / AID

Theory and Application in: Sustainable Energy, Environmental & Urban Planning
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Part I - An introduction to decision problems
A decision problem

- Existence of alternative solutions/decisions/activities/actions

- Multidimensional decision making

  I. Existence of multiple decision criteria (multicriteria decision analysis/aid - MCDA)
  II. Existence of uncertainty in the problem’s data
Decision Making – Decision Aid

- **Decision Making** is a real world process (Social, Political, Economical, Industrial, Environmental, Military, etc.) aiming to:
  - Describe
  - Understand
  - Manage

- **Decision Aid** encompasses two different approaches:
  - Qualitative approach
  - Quantitative approach

✓ Possible decisions?
✓ How to compare them?
✓ Preferences, objectives?
Decision aid/support

Close interaction and discussion between the decision maker (stakeholder, manager, managerial board of a company, investor, ministry, etc.) and the analyst.

Sometimes a person with intermediary role, intervenes between the analyst and the decision maker(s).

Construction of a Mathematical Decision Model, which elicits and suggests the most preferable alternative solution to the decision maker.
Some decision - evaluation problems (1/2)

• Urban planning:
Locating a new plant, refinery, dump, shop

*Decision makers: investors, regional authorities*

• Renewable energy & energy saving:
Investment on the most profitable RES technology for electricity production

Selection of the most cost-effective energy saving actions in a home

*Decision makers: investors, central government, citizens*
Some decision - evaluation problems (2/2)

• Human resources evaluation & management
  Assessment of candidates for recruitment or promotion
  *Decision makers: managerial board of a company*

• Purchasing equipment:
  *Decision makers: practically anybody*

• Project Selection:
  Evaluation of the most promising projects for funding
  *Decision makers: banks, funding bodies, authorities*
Part II - Multicriteria Decision Analysis / Aid philosophy
Decision models Now and Then

**Past**
- Although intuitive every-day decision making perceives and considers multiple objectives, …
- … mathematical decision models included just a single criterion (usually cost or profit)

**Now**
- Intuitive every-day decision making has not changed …
- … however, decision models have incorporated multiple criteria
  - Criteria are weighted based on the preferences of the decision makers
  - Mathematical models attempt to simulate the decision makers’ preference models
Unicriterion vs multicriteria model (1/2)

Unicriterion model:

$$\textit{Optimize } \{g(\alpha) | \alpha \in A\}$$

- $g$ is the evaluation criterion being optimized
- $\alpha$ is a specific alternative action under examination
- $A$ is the set of all the alternative actions which are evaluated

- Mathematically well-stated:
  - Optimal solution
  - Complete ranking of the alternative actions ($A$).

- Socio-economically ill-stated:
  - Single criterion? Not realistic. Real life decision problems always concern multiple criteria
  - Notion of a criterion: perception thresholds, other unique preferences, etc.
### Unicriterion plant location problem

An industrial company wishes to decide on a location to build its new plant

<table>
<thead>
<tr>
<th>Alternative sites</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site 1</td>
<td>5 alternative sites are evaluated based on a single criterion</td>
</tr>
<tr>
<td>Site 2</td>
<td>This criterion (objective function) may be the minimization of investment and operational cost</td>
</tr>
<tr>
<td>Site 3</td>
<td>The model is enriched with all relevant available data and implemented, with a view to selecting the alternative site that bears the minimum cost</td>
</tr>
<tr>
<td>Site 4</td>
<td>A specific site is finally selected among the 5 alternative ones as the cost-optimal</td>
</tr>
<tr>
<td>Site 5</td>
<td></td>
</tr>
</tbody>
</table>
Unicriterion vs multicriteria model (2/2)

Multicriteria model:

\[
\text{Optimize } \{g_1(a), g_2(a), ..., g_k(a) | a \in A\}
\]

\(g_1, g_2, ..., g_k\) are the different evaluation criteria

- Mathematically ill-stated:
  - These problems \textbf{rarely reach an optimal solution}, but a compromise one
  - They do not have a clear mathematical meaning

- Socio-economically well-stated:
  - Closer to real world decision problem
  - Search for a compromise solution
Multicriteria plant location problem

<table>
<thead>
<tr>
<th>Alternative sites</th>
<th>Investment $g_1$ (Million $)</th>
<th>Operational costs $g_2$ (Thousand $)</th>
<th>Environmental impact $g_3$</th>
<th>Social acceptance $g_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site 1</td>
<td>12</td>
<td>179</td>
<td>G</td>
<td>A</td>
</tr>
<tr>
<td>Site 2</td>
<td>15</td>
<td>126</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>Site 3</td>
<td>19</td>
<td>158</td>
<td>VG</td>
<td>G</td>
</tr>
<tr>
<td>Site 4</td>
<td>11</td>
<td>141</td>
<td>VB</td>
<td>B</td>
</tr>
<tr>
<td>Site 5</td>
<td>9</td>
<td>150</td>
<td>G</td>
<td>VB</td>
</tr>
</tbody>
</table>

✓ The multicriterion modeling of the plant location problem also considers environmental and social aspects of the decision.

✓ These are usually essential to be considered, otherwise they may undermine, hinder or downgrade the decision maker’s strategy.

➢ $g_1$ and $g_2$ are **quantitative criteria**, while $g_3$ and $g_4$ are measured on a **qualitative scale** [Very Bad, Bad, Average, Good, Very Good]
The modeling framework in MCDA was proposed by Bernard Roy in the mid 70s. It is based on four stages, which are not followed in a linear way, but they interact; each providing feedback to the other.

It has extensively been used by practitioners and has addressed a vast number of complex multicriteria decision problems.

The MCDA modeling framework is still up to date and is widely practiced.
Stage I – Subject of the decision

A. The subject of the decision is analyzed into a number of different alternative actions
(The whole set of alternative actions, A, to be evaluated is exhaustively defined)

B. Definition of the model’s objective (problematic)

- Problematic $\alpha$: choice (select the most promising alternative)
- Problematic $\beta$: sorting (sort the alternatives in several categories – i.e. good ones, medium ones and bad ones)
- Problematic $\gamma$: ranking (rank all the alternatives from the best to the worst one)
- Problematic $\delta$: description (qualitatively assess the alternatives through description)

More recently, a new fifth problematic was added, namely portfolio selection (select a portfolio of promising alternatives)
Stage II – Consistent family of criteria

Introduction

- The evaluation criteria are defined and modeled, based on this bidirectional cyclic procedure.

- The consistent family of criteria is usually depicted in the form of a tree, see the examples.

Diagram:

- Strict definition of the set of alternative actions
- Analysis of the elementary impact of the set of actions
- Definition of the consistent family of criteria
- Definition of the points of view
- Selection of the evaluation dimensions
Stage II – Consistent family of criteria
Plant location problem

**Problematic**
- Evaluation of the alternative sites for the construction of a plant

**Points of view**
- Costs
- Environmental impact
- Social acceptance

**Criteria**
- Investment cost $g_1$
- Operational costs $g_2$
- Environmental impact $g_3$
- Social acceptance $g_4$
Stage II – Consistent family of criteria
A national natural gas supply problem

- **Problematic**
  - Evaluation of Natural Gas Supply alternatives for a country

- **Points of view**
  - Economics of Supply
  - Security of Supply
  - Cooperativity between countries

- **Criteria**
  - Production Cost ($g_1$)
  - Relative Transfer Cost ($g_2$)
  - Reserves-to-Production ratio ($g_3$)
  - Overall Risk of Corridor ($g_4$)
  - Total Trade ($g_5$)
  - Total Energy Trade ($g_6$)
Stage II – Consistent family of criteria

Definition of a criterion

Each criterion is measured on a preference scale R ($g_i^*$ and $g_i^{**}$ are the worst and best possible values of the criterion $i$).

The criteria take values on a cardinal scale (quantitative scale)

Some are measured using an ordinal scale (numbered discrete scale [1-5], or linguistic scale [poor, low, medium,…, very high], etc.)

Outranking relation

$g(a) > g(b) \iff \alpha Sb$

meaning that alternative action $\alpha$ is preferred to $b$ on criterion $i$.
Stage II – Consistent family of criteria

Criteria properties

3 conditions must apply:

1. **Cohesion / Monotonicity**
2. **Exhaustiveness** (coverage of all the aspects of the decision problem)
3. **Non redundancy** (no overlapping between the criteria)

Each alternative action $a$ is evaluated over all the criteria, and is described by an evaluation vector $g(a)$.

$$g(a) = \left[ g_1(a), g_2(a), \ldots, g_n(a) \right]$$

✓ This evaluation vector is depicted in the multicriteria table (see slide 14)
Stage III – Preference Model

At this stage the mathematical preference model to be pursued is **selected** by the analyst in collaboration with the decision maker and **implemented**.

**Classification of models based on the synthesis of criteria**

- **Compensatory models**
  Compensatory mathematical models can compensate the low scores of alternative actions in certain criteria, through trade-off with the high scores on other criteria.

- **Non Compensatory models**
  These models do not “forgive” the low scores through compensation.

**Theoretical classification of the models**

- **Outranking / Relational methods** *(see Part III)*
- **Functional methods** *(see Part IV)*
- **Analytical / Disaggregation methods** *(see the UTA family methods)*
Stage IV – Decision Support

• The multicriteria mathematical model has been implemented in Stage III.

• In Stage IV the surfacing results are assembled, analyzed and interpreted.

• The analyst assesses the validity and effectiveness of the model, discusses with the DM on the modeling, and explicates some choices within the model.

• This Stage is wholly performed in collaboration with the DM, who may be asked for additional preferential information prior to finalizing the results and decision making.
Part III - Outranking MCDA methods

French – European School
Outranking methods (1/2)

- Outranking methods use outranking relations between two alternative actions: “\(a S b\), \(a\) at least as good as \(b\)”
- The majority principle is followed (vs. unanimity for dominance).
- All actions are compared pairwise.
- Generally, outranking methods approximate better a decision problem
- ELECTRE family methods (Roy, 1968)
- PROMETHEE methods (Brans and Mareschal, 2002)
Outranking methods (2/2)

- An outranking relation $aSb$ is depicted with an arrow from the alternative action $a$ to action $b$.
- $\$\$ denotes the opposite relationship
- $S$ may concern three kind of outranking relations:

$$S = P \cup Q \cup I$$

\[
\begin{align*}
P &= \text{Clear Preference} \\
Q &= \text{Weak preference} \\
I &= \text{Indifference}
\end{align*}
\]

- When two alternative action are proven as incomparable, no outranking relation derives (denoted as $R$)

All relations between alternative actions $a$ and $b$:

- $aPb \iff aSb$ and $b\$a$
- $aIb \iff aSb$ and $bSa$
- $aRb \iff a\$b$ and $b\$a$
The ELECTRE family methods

- ELECTRE I method is used to construct a partial ranking and choose a set of promising alternatives.
- ELECTRE II is used for ranking all the alternatives.
- In ELECTRE III an outranking degree is established, representing an outranking creditability between two alternatives which makes this method more sophisticated.
- ELECTRE IS is used for selection problems, when pseudo-criteria exist.
- ELECTRE methods have been widely used in the literature in all sort of decision problems
- All ELECTRE family methods are however, mathematically ill-structured.

The ELECTRE family methods, despite their amazing realism and countless applications all over the world, do not have the necessary theoretical basis of classification, as the functional models do have.
Part IV - Functional MCDA methods

American School
Functional preference models (1/2)

- Functional preference models evaluate the alternative actions, making use of a **value function** (denoted as \( u \))
- Synthesis of the multiple criteria \( g_1, g_2, ..., g_n \) to a single criterion (global criterion method)
- Evaluation and ranking of the alternatives based on the total score they receive
- They were introduced by Keeney and Raiffa (1976)

\[
g(a) \rightarrow u[g(a)] : \text{The multicriteria vector of alternative action } a \text{ receives a score through a value function } u
\]

\[
g_i^* \text{ και } g_i^* : \text{the best and the worst values of the criterion } i
\]

\[
u[g(a)] > u[g(b)] \quad \text{a is preferred to } b \quad (a > b), \text{ a receives a higher score than } b
\]

\[
u[g(a)] = u[g(b)] \quad \text{a is indifferent to } b \quad (a \sim b)
\]
Step 1: Verification of assumptions about the existence of an analytic value function $u(g_1, g_2, ..., g_n)$ (see Debreu theorem and preference/preferential independence of the criteria in Annex II)

Step 2: The value function $u$ is constructed, based on the preferences of the Decision Maker (criteria trade-offs, etc.)

Step 3: The actions of the set A receive a score and are accordingly ranked (problematic $\gamma$)
The Linear Value Function

Weighted sums method

The linear value function is in essence the weighted sums method.

The value of an action \( a \) is calculated using the following formula:

\[
g(a) = \sum_{i=1}^{n} p_i g_i(a) = p_1 g_1(a) + p_2 g_2(a) + \ldots + p_n g_n(a)
\]

\( p_1, \ p_2, \ldots, \ p_n \) are positive numbers, that constitute the criteria weighting coefficients (criteria trade-offs).

**Attention:** The weighting coefficients are different from the criteria weights (priorities) in the outranking methods. The latter are just numbers that sum to 1, without any physical meaning.
The Linear Value Function

Weighting Coefficients

We assume two actions a and b, whose criteria values are equal on all criteria, except from $g_r$ and $g_i$.

a: $g_1$  $g_2$  ...  $g_r$  ...  $g_i$  ...  $g_n$

b: $g_1$  $g_2$  ...  $g_r - \chi$  ...  $g_i + 1$  ...  $g_n$

Suppose that the decision maker considers that these actions are equivalent ($a \sim b$).

✓ This means that “the trade-off (loss) of a unit of $g_i$ is compensated exactly by $\chi$ units of the criterion $g_r$ and vice versa”

The $p_i/p_r$ ratio of $p_i$ and $p_r$ weights expresses the units of the $g_r$ criterion, that the decision maker agrees to concede, in order to gain one unit on the criterion $g_i$. 

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The Linear Value Function

Summary

Review of the linear method:

The method, although easy and convenient, is unable to manage ordinal criteria. Instead, it requires that all criteria are metric (measured on a continuous quantitative scale).

• The criteria weights have to be stable trade-offs, independent of the amount of the effects of the actions.

• As all other compensatory methods, the linear value function can model the bilateral relationship of incomparability between actions.

Another popular multicriteria functional model is the additive value function (see Annex III for its description)
Part V - MCDA application in energy planning

Siskos et al. 2014
An overview

Power generation investments from RES

- Sufficient geological and climatic potential
- Dynamic nature that needs thorough and careful planning
- Selection of optimal investment in the present moment
- Multidimensional decision making problem

Research Objective

Selection of an optimal investment for an individual investor

- Critical review on relevant published studies and surveys
- Modeling the selection of the optimal power generation from RES investment
- Existence of both quantitative and qualitative criteria
- Implementation of the most suitable evaluation framework
- Support the decision of a private investor
Problem Description

Subject of the Decision: Selection of the optimal investment for power generation from RES in the range of 10 MW (Problematic α)

Set of alternative actions

a) Photovoltaic station (Interconnected)
b) Photovoltaic station (Not interconnected)
c) Wind plant (Interconnected)
d) Wind plant (Not interconnected)
e) Small hydroelectric plant
f) Solar thermal plant (Interconnected)
g) Solar thermal plant (Not interconnected)
h) Geothermal power plant
i) Biomass power plant
Criteria Modeling

Multicriteria evaluation of power generation investments from RES

- Social
  - Jobs creation
  - Social acceptance
  - Additional social benefits
- Economic
  - Investment cost
  - Operational and maintenance cost
  - Electricity selling price
  - % Subsidy
- Technical
  - Efficiency
  - Compatibility
  - Reliability
  - Know how
  - Operational safety
  - Preparation time
  - Life Cycle
- Environmental
  - Effect on soil
  - Effect on water
  - Noise pollution
  - Landscape degradation
  - Required area
- Political
  - Penetration margin
  - Stability & policy bureaucracy

Social criterion ($g_1$)
Cost criterion ($g_2$)
Revenue criterion ($g_3$)
Effective operation ($g_4$)
Expertise ($g_5$)
Project cycle ($g_6$)
Environmental criterion ($g_7$)
Required area ($g_8$)
Political support ($g_9$)
## Assignment of values

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$g_1$ (1-5)</th>
<th>$g_2$ (M €/ MW)</th>
<th>$g_3$ (€/MWh)</th>
<th>$g_4$ (1-5)</th>
<th>$g_5$ (1-5)</th>
<th>$g_6$ (Years)</th>
<th>$g_7$ (1-5)</th>
<th>$g_8$ (W/m²)</th>
<th>$g_9$ (1-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4</td>
<td>2</td>
<td>406</td>
<td>3</td>
<td>5</td>
<td>18</td>
<td>4</td>
<td>15</td>
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<td>15</td>
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<td>1</td>
<td>48</td>
<td>3</td>
<td>60</td>
<td>3</td>
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<td>i</td>
<td>3</td>
<td>18.1</td>
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<td>2</td>
<td>38</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

**Multicriteria table** depicting the score of each alternative on the 9 criteria
Methodological Frame

Implementation of a synergy of MCDA methods

- Evaluation of the RES alternative through the Electre I outranking method
  - Selection of the most preferable alternative action
- Weights inference through the Simos method
  - Huge diversification among the 9 criteria
  - Facilitation of the DM to indirectly assign weights to the criteria

- The Simos method (Simos, 1990, Figueira and Roy 2002)
  - Criteria cards
  - White cards
  - Fasteners
  - Hierarchy by the decision maker
Weights elicitation via the Simos procedure

**SIMOS results**

- $w_3 = 0.224$
- $w_2 = 0.172$
- $w_5 = 0.132$
- $w_6 = 0.132$
- $w_4 = 0.112$
- $w_7 = 0.072$
- $w_1 = 0.072$
- $w_9 = 0.052$
- $w_8 = 0.032$

Linear mathematical problem solution, maximizing $w_3$

Descending order of importance
Implementation of ELECTRE I

- Assessment of Veto Thresholds
  Definition of the veto values after dialogue with the DM

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Veto Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost criterion (Millions €/MW)</td>
<td>10</td>
</tr>
<tr>
<td>Revenue criterion (€/MWh)</td>
<td>170</td>
</tr>
<tr>
<td>Effective operation</td>
<td>{2,5}</td>
</tr>
<tr>
<td>Expertise</td>
<td>{1,5}</td>
</tr>
</tbody>
</table>

- Concordance check C(a,b)
  (concordance index $s = 0.7$ – defined by the analyst)

- Discordance check
Outranking relations are depicted in the graph using arrows.

The 4 alternative actions, which form the core of the outranking graph, are the most promising ones, since they are not outranked by any alternative.

Some pairs of actions are incomparable (bRe, bRi, eRi)
Among the four most promising investments, one must be selected. The potential investor should be supported to select the most preferable one. (Stage IV)

- Rejection of the alternative b after observing its criteria scores: X
- Further insight and dialogue on the alternative I, which is rejected: X
- Further insight and dialogue on the alternatives e and α: √

Interactive comparison of a-e:

Decision Maker: In overall I prefer a to e

Best alternative: Photovoltaic station (Interconnected)
Graphical Illustration of Decision Support

- Photovoltaic Station (Interconnected)
- Photovoltaic Station (Not interconnected)
- Small hydroelectric plant
- Biomass power plant

First analysis of the results
Approval by the DM
Decision Making

Electre

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Part VI - Bibliography
Bibliography on MCDA theory


Bibliography on MCDA theory


Bibliography on MCDA applications


Bibliography on MCDA applications


Annex I - The ELECTRE I method
ELECTRE I (Roy, 1968)

ELECTRE I is a selection outranking method (problematic α), on the basis of the majority rule, making use of veto thresholds, which render some alternatives incomparable.

- **Criteria weights** \( p_i \), set priorities between the criteria, and are extracted by the DM. They are normalized using the formula:
  \[
  \sum_{i=1}^{n} p_i = 1
  \]

- **Agreement/Concordance threshold s:**
  A number inferred by the analyst ranging from 0.5 to 1. It is used in the agreement control phase of the method.

- **Veto thresholds** \( (v_1, v_2, ..., v_n) \):
  Each corresponds to a specific criterion and controls big variations between the values of the actions on the criterion. They are also extracted by the DM. These are accounted during the disagreement/discordance control.
ELECTRE I (Roy, 1968)

ELECTRE I requires alternatives and criteria to be specified, and uses the same data of the decision table, namely $g_i(j)$ and $p_i$.

For an ordered pair of alternatives $(a,b)$, the agreement/concordance index is the sum of all the weights for those criteria, where the performance score of $a$ is least as high as that of $b$, i.e.

$$c_{ab} = \sum_{i: g_i(a) \geq g_i(b)} w_i \quad a, b = 1, ..., n, \quad a \neq b$$

ELECTRE I finally results to an outranking graph ($\Pi$), illustrating all outranking relations and incomparabilities. The alternatives, which are not outranked by any other, form the core of the graph, after considering the disagreement control.
Flowchart of ELECTRE I

1. Problematic α
2. Weights p
3. Threshold s
4. Threshold of indifference - preference
5. Multicriteria Evaluation Table
6. Discrete action set A
7. Agreement Control
8. Disagreement Control
9. Veto v
10. Relationship/superiority graph S
11. Circuit management
12. Core Π
13. Decision Support
14. Sensitivity Analysis
Annex II - Debreu theorem & Preference independence
Debreu theorem

**Debreu Theorem:** The decision model of a person is a linear value function when, for this person, the trade-offs $s_{ir}^g = \frac{p_i}{p_r}$ between the criteria $(g_i, g_r)$ are:

- **Independent** of the values that the other criteria $F - \{g_i, g_r\}$ receive, and
- **Constant**

Preference independence

The concept that dominates the theory of the additive value model is the preference/preferential independence of the criteria. The strict definition is given below:

**Definition:** A pair of criteria $(g_i, g_j)$ is preferably independent to all the other criteria $F - (g_i, g_j)$, when the trade-offs values between the criteria $g_i, g_j$ are not dependent on the values of the other criteria.
Annex III - The additive value function
The Additive Value Function

Mathematical formulation

An additive value function is more elaborative than the linear one, and is defined as follows:

\[ u(g) = \sum_{i=1}^{n} p_i u_i(g_i) \]

\[ u_i(g_i) = 0, \quad u_i(g_i^*) = 1, \quad 0 \leq u_i(g_i) \leq 1 \quad \forall i \]

\[ \sum_{i=1}^{n} p_i = 1 \]

- \( u_i(g_i) \), \( i = 1, 2, \ldots, n \) are non-decreasing marginal value functions, normalized between 0 and 1.

- \( g_i \) and \( g_i^* \) the worst and the best level of the criterion scale, and

- \( p_i \), \( i = 1, 2, \ldots, n \) the weight coefficients of the marginal value functions, whose sum is equal to 1.
The Additive Value Function

Marginal Value Functions

- The marginal value functions are used in order to express a **possible variation of importance** along the scale of each criterion separately.
- After verifying the preference independence of the criteria, the analyst is entitled to work for the construction of each function $u_i(g_i)$ separately, considering the preferences of the decision maker.
The Additive Value Function

Summary and review

1. The additive value model is wider and more sophisticated than the linear, since the latter derives from the first, if we set \( u_i(g_i) = g_i \), \( i = 1, 2, \ldots, n \).
   Consequently, the weighting coefficients \( p_i \) of the additive value model are the trade-offs between each pair of \( u_i(g_i) \), which, are constant.

2. The additive value model can be also applied when ordinal criteria exist, which are often encountered in practice.

3. The validity assumptions of the additive model are less strict than these of the linear model.

The sole requirement of this model is that **the trade-offs between the criteria are independent of the other criteria, not strictly stable.** (see preference independence in Annex II)